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# The forgotten harmonical science of the Bible

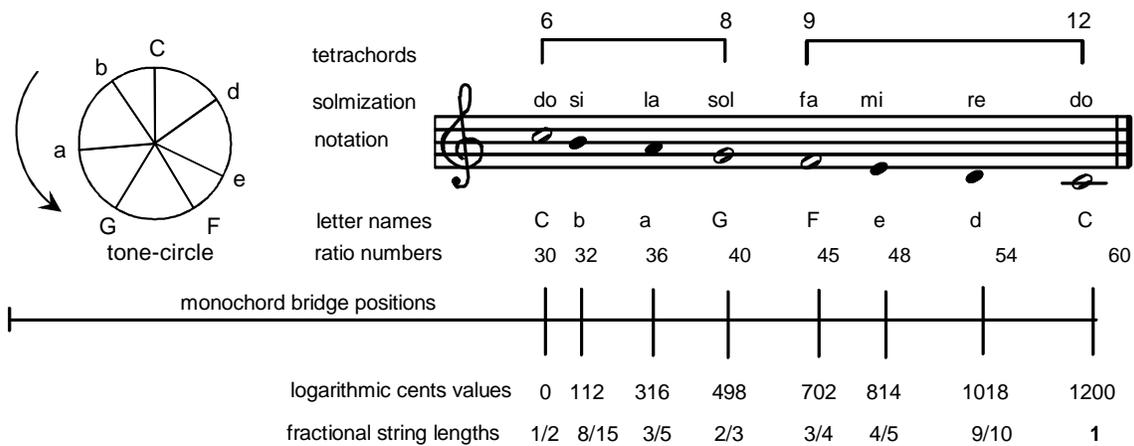
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## 1. Introduction.

Biblical creation “by measure, number and weight”<sup>1</sup> required God to possess a fluency in arithmetic not always shared by the faithful. And so Bible arithmetic of the first millennium BC eventually became incomprehensible. This is a progress report on its decoding.<sup>2,3</sup>

Commenting on *Genesis* early in the first century AD, and with the benefit of a Greek education, Philo of Alexandria recognized that tribal narrative carries symbolic meaning in harmonical theory.<sup>4</sup> But creation “by the word” could not have entailed any time whatever; the creation “week” is authorial allegory. What God required logically was the set of first seven integers--1, 2, 3, 4, 5, 6, 7. And so Philo concluded that Moses, putative author of the first five books of the Bible--*Torah* (“Law”) to Jews and the *Pentateuch* to Christians--must have understood tuning theory long before Pythagoras brought it home to Greece.<sup>5</sup> Figure 1 summarizes complementary basic insights.

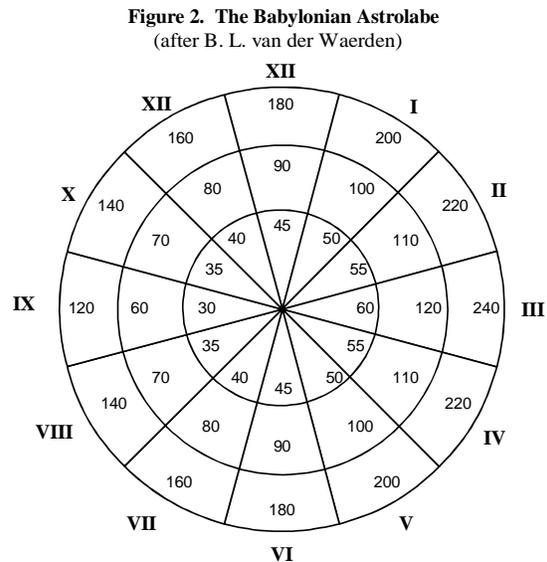
Figure I. Equivalent representations of the basic musical scale.



The base 60 arithmetic of ancient Sumer and Babylon, whatever the reason for its adoption, naturally favored this particular tuning of the scale. Up to 60 the ratio numbers agree with base 10. Thus harmonic theory may have developed early in the third millennium BC when a great variety of harps, lyres, pipes and other instruments attest to considerable empirical experience with quantification on wind and string instruments.

Today much of the astronomy, arithmetic, and music attributed to Classical Greece is documented to Semitic Babylon in the second millennium B.C.<sup>6,7,8</sup> Mesopotamian fluency in calculation--in the age of Abraham, Isaac, and Jacob--already was 3000 years ahead of 16th century AD Europe. Babylonian exile in the sixth century BC made accessible to the Jews anything not already known.

With Philo's help I am reading Divine *prescience* as pre-scientific musical insight encoded in tribal mythology.<sup>9</sup> Biblical emphasis on twelve sons as eponymous ancestors of twelve tribes who build an altar of twelve stones concerns twelve idealized "boundary markers" in a cyclic octave needed for Davidic musicology. The pattern was long symbolized in the concentric circles of the Babylonian *astrolabe*, adjusted monthly to correlate the watches with the varying lengths of day and night.<sup>10</sup> Figure 2 strips all star data from van der Waerden's reconstruction, and converts his base-60 water-clock weights (for full watches in the outer circle, and half and quarter watches in the inner circles) to base 10 arithmetic. The astrolabe's naked geometry and simple arithmetical doubling expose the *idealist* mind set which guided the evolution of Chaldaean sciences--converted to priestly ritual by Jewish ingenuity. Twelve ideal months of 30 fictitious days were superimposed on the heavens, and the ratio of longest night to shortest day, known to be about 3:2, was computed as 2:1, so that only music offered a "manipulable" example which conformed to these *rounded* measures.<sup>11</sup>



Concentric circles anticipate Ezekiel's "wheel in a wheel" as the throne of heaven. Within each circle maxima and minima water clock weights of 2:1 anticipate the ratio of cyclic octaves. But equal weight differences between successive months (reversing at the solstices in months III and IX) had to give way to proportional differences between successive semitones when this geometry was applied to music. Rational tonal arithmetic, cleverly mimed by Ezekiel, could anticipate this conceptual equality only via a slight but cumulative excess or deficiency, for in a cyclic octave of ratio 1:2 all equal divisions are defined by irrationals. Thus the Holy Land of a spiritual Israel had to be conquered conceptually in intricate warfare between the *excess* of primordial "giants" (products of 3) and the *deficiency* of human "weaklings" (products of 5) among *rational* numbers, and "weaklings" won only with Divine help in circumventing the lack of *real* number. Bible narrative brilliantly allegorizes every aspect of *Diophantine approximation* to modern Equal Temperament, and it does so with exhausting respect for numerical detail--making the Bible a priceless repository of tuning lore and its

elementary number theory. The “unhewn” stones of Jewish altars are *integers*, meaning the *natural* or *counting numbers* to which harmonical theory normally was restricted, although its calculation demanded great fluency with reciprocal fractions. From the perspective of any reference pitch all integers except 2<sup>n</sup> necessarily “sinned” by “missing the mark” to some degree because octave doubling imposed, *a priori*, a universal matrix (“womb”) tied to integral powers of 2. Problems arose immediately with division into 2 equal parts (requiring the square root of 2) and 3 parts (requiring the cube root of 2).

*Sensory intuition* always fails at some level of arithmetical subtlety where *least noticeable differences* create a Platonic “no man’s land” of uncertainty. Greek, Jewish, and Chinese cultures are unanimous in accepting the *comma* of 80:81 as its convenient normative value. It is the difference between a “giant” whole tone of 8:9 (worth 204 *cents* in modern logarithmic measure) and a “human weakling” of 9:10 (worth only 190). They are approximations to *the sixth root of 2* worth 200 *cents*, the value necessary to divide an octave 1:2 into six equal parts. How Davidic tuning theory reconciles this conflict becomes the central focus of Bible allegory. And in the sixth century BC only God could have solved this problem numerically--although any geometer could map results to his own satisfaction for the astrolabe pointed the way.

For musician/philosophers of Philo’s temperament, tuning theory may always have been a contest in the soul between the potential tyranny of *masculine* intellection, considered mankind’s very highest power, and the relative benevolence of our *feminine* sensorium, where *least noticeable differences* create some measure of perceptual tolerance. *Wisdom* required a congenial mating between our own masculine concepts and feminine percepts, and sometimes rewarded it with the experience of transcendent beauty in “out of the body” adventures like Philo enjoyed when listening to the antiphonal singing of segregated sexes in his Alexandrine synagogue. I am trying here to articulate Bible harmonics in Philo’s spirit while paying closer attention to its computational logic.

Philo’s insight proves to decode most (though not yet all) Bible numerology. And that decoding exposes the poetic metaphors in which the Jews embodied algebra. As we gain a stronger hold on this forgotten Bible arithmetic, the literary genius of authorial allegory reveals its power to encode structure and meaning on two parallel levels: 1) in an esoteric arithmetic probably never understood except by experts in computation, but therefore needing only cryptic allusion; and 2) in homely parables which make symbolic meaning clear even to the illiterate, and thus protect priestly allegory against any accusation of elitism. I am concerned here mainly with the forgotten tonal logic in biblical numerology, employed with perfect discipline by Bible authors from the first page of *Genesis* to the last page of *Revelation*--where Jewish playfulness fades into Christian dogma

## 2. What it means to be “Chosen.”

YHWH (𐤅𐤇𐤍𐤅 in Hebrew), the tribal God whose unpronounceable 4-letter name appears about 6000 times (usually rendered as LORD in English), emphasizes the principle of *smallest integers* which keeps the decoding of Bible arithmetic forever

possible across any chasm of time, space, or language. He orders Moses to remind the Chosen People that,

The LORD did not set his love upon you, nor chose you, because ye were more in number than any people--for ye were the fewest of all peoples--but because the LORD loved you ... (Deut 7:7-8).<sup>12</sup>

As universal *model builder* the Jewish God favors the principle of maximum economy, meaning the *fewest integers* possible in each and every circumstance. The creation “week” ensures that primes larger than 7 play no defining role, and that cyclic repetition begins at the octave. David as musician/king and designer of the Temple focuses attention more directly on music.

The equal responsibilities assigned to 12 tribes of radically different sizes already points toward 12-tone equal division as an idealist norm. (Ezekiel ignores geography in distributing land equally between the tribes.) Thus--although at first it may seem wildly anachronistic--I treat Davidic tunings as approximations to equal temperament, and try to ignore everything but the recovery of authorial tonal models. Only the Bible’s own arithmetic and its own statements about it can be trusted. No later commentator has ever equaled Philo’s insight. Although his own arithmetic often proves inadequate, his clues remain priceless. One difficulty is that Bible authors were far more expert with this matrix arithmetic than any later scholars have proved to be.<sup>13</sup> They coerce us into adopting an obsolete methodology in order to appreciate how fully their musical wisdom actually anticipates our own.

Moses is taught by Jethro, his Kenite father-in-law, to distinguish between “great” and “small” matters, and to choose “rulers of thousands, of hundreds, of fifties, and tens” to decide the “small,” preserving his own energy for “every great matter.”<sup>14</sup> Jethro’s “great and small” parallels Plato’s “dyad of the great and the small.”<sup>15</sup> This metaphor (a description attributed to Aristotle) alludes to the *reciprocal* meanings of integers as *multiples and submultiples* within the cyclic “dyad” of the musical octave 1:2. The *Chosen People* are expected to remain loyal under any reversal of fortune (arithmetical reciprocation), as illustrated by Abram in offering nephew Lot first choice of the Holy Land, declaring himself indifferent to right and left, dry hills or well-watered valleys, and as dramatized by Job, whose patient suffering is compensated by a doubling of his former wealth.

The logical discipline controlling Bible arithmetic warns a musical cryptographer that any reduction in Bible numerology destroys something of authorial interest--and that any model *octave double* 1:2 however greatly multiplied also enjoys a “small” set of reciprocals lying between 1 and 1/2. Leading digits enjoy “followers” of “thousands, of hundreds, of fifties, and tens” because the decimal point is not yet invented, and so ratio numbers are inflated to avoid fractions in the same “floating place value” decimal arithmetic which pocket calculators still employ. Authors and redactors are disciplined by these principles while Masoretic editing is completed in the first millennium AD, and the chapter and verse numberings which it introduced sometimes seem artificially

contrived to hint at the tonal algebra within their texts. The creation week at the opening of *Genesis* is the very first example of extreme *Yahwist* economy. Its Canaanite emphasis on “7 days” (apparently borrowed from Ugarit) is a reduction of the Babylonian/Greek Holy Ten-ness which also leaves many echoes in Bible texts, notably in the Ten Commandments. The hated biblical Baal is “God 10” (Marduk) in Babylon, and it is tempting to suppose that the Jews are borrowing their conqueror’s science to parody their conqueror’s religion--and daring to do it in exile in sixth century BC Babylon, so that a certain reticence to explain arithmetic detail more clearly may have been politically expedient. In any case, *allusion* is the preferred Jewish literary style. Irony abounds, wit stays near the surface, and so all interpretation involves risk.

### 3. Bible harmonics as prelude to Jewish philosophy.

Creatures conceived during the seven days of the creation week (like all fruits containing their own seeds) are bid to “be fruitful and multiply” because multiplication is the only operation relevant to ratio theory. And 7 is systematically expanded into the 7-year priestly cycle of  $7 \times 360 = 2,520$  days, or  $2 \times 2,520 = 5,040$  “days plus nights,” enlarging the tonal system at each step. This early example of *combinatorics*, disciplined by a *set theory* borrowed from music, thus proves most satisfactory on the monochord where all detail can be integrated.

Both Ezekiel and Plato project their arithmetic into similar concentric circles, “a wheel in a wheel,” functioning as the throne of an idealized heaven. Plato’s analysis of 5,040 fits many of Ezekiel’s metaphors and thus facilitates decoding the *sameness and difference* between nascent Greek science and traditional Jewish wisdom. *This is the cross-cultural ambiance in which Philo was educated and about which he wrote with equal passion for Greek learning and for his own religion, which shared the same models.* The music of the synagogue embodied their union and freed his soul to roam where it would. The two musical modes decoded from Bible numerology have proved to be associated historically with *the mode of the Torah* (Greek Dorian) and *the mode of the Prophets* (Greek Phrygian) in ways Philo helps us understand; they are the two modes Plato admitted in model cities.<sup>16,17</sup>

The importance of the priestly 7-year calendrical cycle is emphasized in *Ezekiel* 39:10 where God insists that after his destruction of Israel’s enemies the country will have no “need to take wood out of the field or cut down any out of the forests” for a period of seven years, “for they will make their fires of the weapons” of warfare. I analyze the tonal content in 5,040 “days plus nights” as furnishing Jewish “weapons” of spiritual warfare not merely on this circumstantial biblical evidence but because this also follows Jewish philosophical precedent. In *Sepher Yezirah*, “A Book on Creation” written perhaps about the fourth century AD and considered by some to be the founding document in Jewish philosophy, *factorial seven* meaning  $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5,040$  is decomposed with a cryptic competence which Philo did not possess personally, and so I fit his musicology into its schema. In this later document the Jews have tired of musical games and are putting harmonics behind them in favor of more general philosophical concerns. Thus it was the honest urge toward more general meaning which eventually

helped Judaism to forget its debts to music. *Sepher Yezirah* elegantly directs attention to the systematic expansion of the harmonical system through successive *factorials*.

Two stones build two houses, three stones build six houses, four build twenty-four houses, five build one hundred and twenty houses, six build seven hundred and twenty houses and seven build five thousand and forty houses. From thence further go and reckon what the mouth cannot express and the ear cannot hear.<sup>18</sup>

Factorial 7 leads us to “what the mouth cannot express and the ear cannot hear.” Today we are generally introduced to quantified tone ratios via a harmonic series on C. I invent a *Philonic preview* to ease readers into the past.

#### 4. The creation week as an harmonic series.

The conception of Israel as *Jeshurun* (“Upright”) and symbolized by “male intellection” correlates with integers as divisors of string length and thus with the numbering of an harmonical series.<sup>19</sup> On the monochord (real or imagined) the *even* integers 2:4:8 etc. reaffirm the *reference pitch class* (whatever it may be) to infinity as they establish the cyclic octave as a *matrix of doubles*,<sup>20</sup> functioning as “cosmological mother.” But the ambiguity of “Day 1” in consisting of “evening and morning” makes creation already a *union of opposites*. The harmonic series shown in Figure 3 as an ascending sequence of successively smaller intervals must be imagined as *inverted* under reciprocation when the integers function as *multiples of string length*.

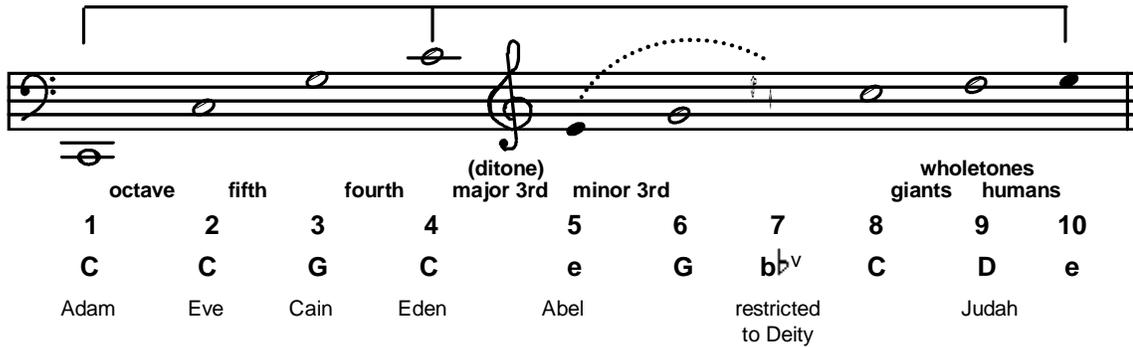
Odd integers introduce new pitch classes into the matrix and therefore are “male.” Numerical doubling reaffirms the same pitch class and therefore is a sign of God’s approval.<sup>21</sup> Relations are mythologized in several alternate ways. Eve as “Adam’s rib” lies at the mid-point on the monochord under division. Cain as first odd integer is “first son,” the prime number 3, destined to be a “wanderer on the earth.” (His locus on the monochord depends upon our viewpoint.) He is the ancestor of both Jubal, “father of all those who play the lyre and pipe,” and of Tubal-cain, “forger of all instruments of bronze and iron.” His sister is Naamah (“Song”).<sup>22</sup>

Jewish monotheism is aware of the basic theorem of arithmetic that all integers can be expressed uniquely as the product of primes, and so arithmetical doubling (which leaves pitch class unchanged) becomes quite generally “a symbol of God’s approval”—but the female 2 now is fully integrated into the masculine Godhead. Primes larger than 7 play no role as tone generators.<sup>23</sup> Doubles always define the same pitch classes, eventually giving rise to the notion, exemplified in Eve’s creation from Adam’s rib, that metaphorical woman is only “half-a-man” and therefore requires double her husband’s value to be a proper mate. Cain (3) kills younger brother Abel (5) and when questioned about the latter’s whereabouts answers, “Am I my brother’s keeper?” He is, for 5 functions conceptually as the arithmetic mean in 2:3=4:6. And  $2 \times 3 = 6$  becomes the mean between Abel (5) and God (7). Mythological metaphors are never entirely abandoned.

Because the integral powers of prime numbers never agree except at the *zero power* equated with “1,” tonal creationism is bedeviled with incommensurables for which God shares responsibility by providing one or more “saviors” at strategic moments. Our 12

*equal-tempered* semitones function as idealized “boundary markers,” but smaller intervals also were admitted into ancient tuning theory, for the string is treated as a continuum of pitch. A conventional harmonic series on C illustrates the important ratios.

**Figure 3. An imagined "Philonic" preview of Bible harmonical cosmology.**



Eden = 1:2::2:4.

The senarius of six "working days."

7 days of creation as idealist harmonical "material."

The tonal matrix as 2x2x2 = 8 and the first perfect cube.

3x3 = 9 is "most warlike" among the first ten integers, eventually "Judah."

Babylonian (Marduk, the biblical Baal) and Greek holy "Ten-ness," here co-opted.

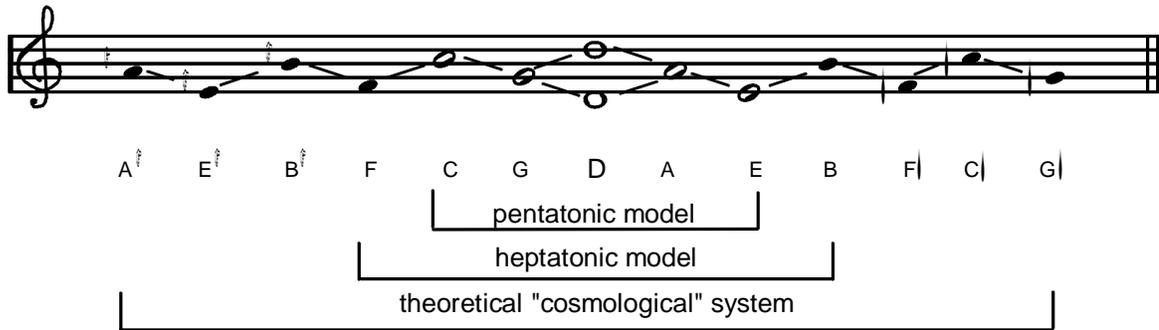
Bible texts cite numerically significant clues and so give the appearance of being numerologically inspired--which they are, but within a strict tonal logic, massively documented, and still awaiting understanding.

<b>Table 1. Some useful modern examples of the manipulation of ratios.</b>	
Ancient matrix arithmetic provided fast and easy similar results via “pebble” patterns.	
Ratios are expressed here as fractions compounded by multiplication.	Irrationals were approximated and excess and deficiency were computed and compensated.
2:3 plus 3:4 = 3/2 x 4/3 = 12/6 = 2	7/5 = 1.4 approximates the square root of 2
4:5 plus 5:6 = 5/4 x 6/5 = 30/20 = 3:2	5/4 = 1.25 approximates the cube root of 2
9/8 minus 9/10 = 9/8 x 9/10 = 81:80 ( “comma”)	9/8 = 1.125 approximates the sixth root of 2
<b>Fractions were avoided via prior decimal expansions by 10s, 50s, 100s, 1000s, etc.</b>	

The prime number 3 subdivides its octave (2:4) into the musical fifth 2:3 and a complementary fourth 3:4, and these ratios can be reiterated to produce Spiral Fifths

tuning. Each new pitch class requires definition by a higher power of 3. The twelfth musical fifth (defining the thirteenth pitch class) requires  $3^{12}=531,441$ , just slightly beyond  $2^{19}=524,288$  to which the reference unit must be doubled for direct comparison. This micro-interval of a *comma* is too small to be useful, and so the 1st and 13th tones in the sequence must never appear together. Old Testament arithmetic is cleverly contrived to define the 13 pitches with only 12 numbers, without going beyond  $3^{11}=177,147$  defining the twelfth pitch class. But even this limit is irrelevant to the ear, which cannot distinguish pitches aurally beyond the second or third defining digit. Jacob, under Yahweh's protection, dies at 147 after fathering twelve sons in a more economical Just tuning to mock the inaudible "tail" in 177,147, required conceptually but useless perceptually. Figure 4 locates the tones of the "spiral" within a model octave on D, center of symmetry in modern notation.

Figure 4. The "Great Serpent" spiral of musical fifths and fourths.



The simplest rule for arithmetizing this sequence, applicable to any number of tones, is "add or subtract one-third."<sup>24</sup> On the monochord, adding one-third locates a falling fourth of 3:4; subtracting one-third locates a rising fifth of 2:3. (The rule can be applied arithmetically, however, only by beginning with the largest power of 3 needed, always one less than the number of pitch classes to be included.)<sup>25</sup>

In Bible metaphor these children of "3:2" are *Nephilim*, "giant demigods" or Angels (each worth about 702 *cents* instead of 700), and their cyclic excess adds up to about  $2 \times 12 = 24$  *cents*. For some authors they symbolize the *Rephaim*, the aboriginal giants of Canaan, Edom, Moab, and Ammon, from whom the Holy Land must be wrested. Or the *Anakim* (sons of Anak), so named from a Hebrew root meaning "neck" or "necklace," for they do indeed divide the cyclic octave into a metaphorical necklace of twelve nearly equal parts.<sup>26</sup> By contrast, Jewish giants are imagined as ritually "circumcised," and Jewish folklore even allows certain heroes to be "born circumcised."<sup>27</sup>

The prime number 5 can be conceived as 3+2 by addition, producing the "weaker" *male* "fivers" (humans) whom God loves even more than Angels and Giants--and for the reason that they lead to a great reduction in numerosity and, for musicians like Philo, to more numerous consonances with other pitches.<sup>28</sup> In a musical context, 5 subdivides its musical fifth (at  $2:3 = 4:6$  in Figure 3) into a pure major third of 4:5 (worth only 386 *cents*, a severe cyclic deficiency of 14 *cents*) and a complementary minor third of 5:6 in a *Just* tuning system which is favored by base 60 notation ( $3 \times 4 \times 5 = 60$ ). This Just ditone

allows 12-tone tuning theory to be reduced from six digits to three digits in ways shown later, and permits the favored mode to be defined in only two digits.

Philo describes the odd, male integer 9 ( $=3^2$ ) as “most warlike” among the first ten. It defines an oversized whole-tone of 8:9 (worth about 204 cents), standard in Spiral Fifths tuning, and measuring the octave 1:2 as just slightly less than six whole-tones. It leads to a *ditone* (i.e., a major third) of  $(8/9)^2 = 64:81$  of 408 cents. In Just tuning one Spiral Fifth giant of 8:9 is coupled with a Just weakling of 9:10 (worth only about 190 cents) in the ditone of 4:5, worth only 386 cents. These options must be selected with great care or the tuning system rapidly degenerates.

The prime number 7, reserved to Deity, offers the simplest approximation to the square root of 2 in the ratio 7:5. Its worth can be tested in the proportion 35:49::50:70, where its cumulative deficiency of 49:50 is precisely centered in the octave, “missing the mark” by an excess or deficiency of only one part in a hundred. Here is the “lost sheep” a Savior will rescue later. Davidic tuning offers several better approximations, and so “7” is not needed as a “working” number. Here it maps the limits of Divine Tolerance. “Priestly” allegorists manipulate their arithmetic to handle this problem in several different ways. By 1800-1600 BC the Babylonians already possessed a square root of 2 approximation correct to five decimal places,<sup>29</sup> sufficient for *equal temperament*, and the diagonal of any square solves the problem perfectly via geometry, so the Jews are playing consciously and expertly, albeit mythologically, with a well-understood problem. We know this subject as *Diophantine approximation* (named for an Alexandrine arithmetician of the third century AD). The full range of Jewish insight, however, is exposed only at factorial  $7=5,040$  (i.e.,  $7!$ ), and after the names of God are understood.

Figure 5. The Septimal Comma  
35:49::50:70

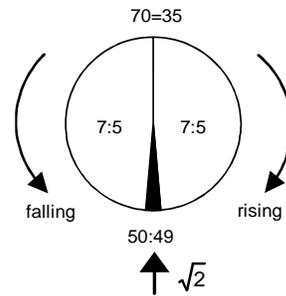
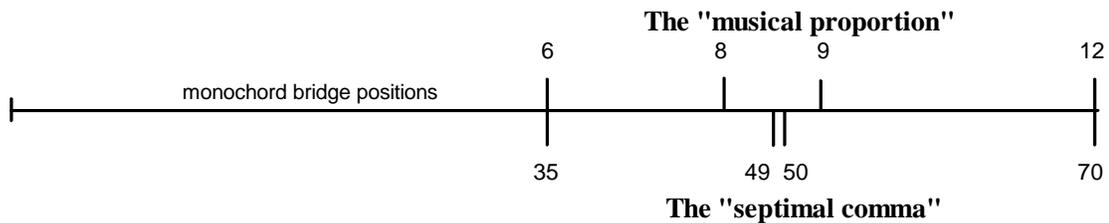


Figure 5a. The normative range of Divine Tolerance as measured by the septimal comma.



Against the fixed reference of the musical proportion 6:8::9:12, the “septimal comma” of 49:50 within the proportion 35:49::50:70 establishes the normative limits of

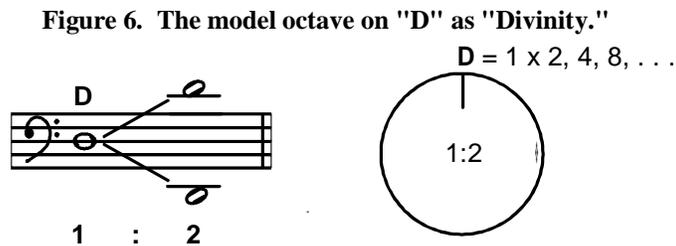
Divine Tolerance (in this case of missing the square root of 2 by no more than an excess or deficiency of one-percent). Several Bible approximations fall "naturally" within this limit, thus 7 is never needed as "working day," else it would inflate numerosity grotesquely by a factor of 7.

**5. The meaning of "One."**

God claims to be both "beginning and end," and this *verbal* unity obviously embraces both "whole" and "part." The avoidance of fractions by *least common multiples* allows any integer factorable by 2, 3, 5, and/or 7 to function locally as *God surrogate* in some harmonic model. But while recognizing all integers as multiples of his own unity--rendering all other Deities obsolete--only numerical doubling via *powers of 2* keeps the reference pitch class constant. Thus God plays one role as judge of the whole--seated "above the circle of the earth" on any *numerical index* (largest integer) required--and a usually quite distinct role as the creative Demiurge, the small "1" from which all integers emanate as multiples, exposed by some power of 2 whose locus we must discover for ourselves. Israel's enemies mocked its "blind Creator" for not anticipating the difficulties which he makes for himself, but Bible authors rejoice in these problems of Diophantine approximation and solve them to their own satisfaction with great imagination. Thought is now moving at a higher level of abstraction than in contemporary Babylonian mythology.<sup>30</sup> Isaac will emerge as the "first-born" Jew in the *septimal comma* between 49: and 50--as a clever square root of 2 approximation. He will live 180 years, for the circle is conceived as 360 artificial "days" or "degrees." His mother is the 90-year-old Sarah, who dies at 127=1+2+4+8+16+32+64 to help us identify him (the sum of these successive powers of 2 is always one less than the next "double" in the sequence). God stage manages solutions in such a way that we never need to multiply by 7 except to expose his subtlety.

**6. Factorial 2 (2!) defines the octave matrix.**

Factorial 2, conceived as 1x2=2, seems trivial until we realize that integers have double meanings as "great" and "small," (i.e., *multiples* and *submultiples*), a biblical motif apparently inherited from Ugarit and shared with Greece.



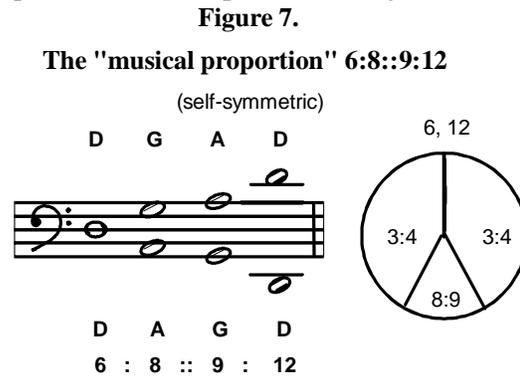
Multiplication and division by 2 has no effect on pitch class.

The separation of light from darkness on Day 1 ("evening" and "morning") is followed by the establishment of a "firmament" on Day 2 when the *octave double* 1:2

provides “two houses” for the same *pitch class*, meaning at “1” and at “2” (or “1/2”) on the string. By functioning as both *half* and *double*, “2” encourages Divinity to define himself as “beginning and end,” or even as “beginning, middle, and end.”

**7. Factorial 3 (3!) integrates the musical fifth 2:3.**

Factorial 3 as  $1 \times 2 \times 3 = 6$  “marries” the first two primes via multiplication to symbolize the mankind (male 3 and female 2) created “in the image of God” on the sixth day of Creation. But “3” automatically leaves an *octave complement* of the musical fourth 3:4 in the ratio numbers 2:3:4 which apply both to a higher and a lower octave, shown here as D:A:D rising and D:G:D falling. When brought together within the “D” octave the new tones, A and G, become “twin sons” via the prime number 3. The ratio 2:3 can be taken from both ends of the octave 1:2 only when it is expanded to 6:12. Musical fifths of 2:3 then overlap at 6:9 and 8:12 so that musical fourths of 3:4 result at 6:8 and 9:12, and the whole tone of 8:9 emerges as the difference between a fifth and a fourth in the self-symmetric *musical proportion* within which the scale is developed.



David’s name in Hebrew is  $\aleph \blacklozenge \aleph$  symbolizing the integers 4.6.4, so that letters and numbers mirror the reciprocity of musical fifths ( $2:3=3:2$  in both directions). In Moses’ metaphors, by shifting perspective from 2 to 6 we have traveled “three days into the wilderness” of ratio theory to worship the God of Jewish rationalism.<sup>31</sup>

**8. Factorial 4 (4!) “seals” the fixed tones which frame tetrachords across 2 octaves.**

Factorial 4 meaning  $1 \times 2 \times 3 \times 4 = 24$  “seals” the *musical proportion* in Figure 7 over a double octave--6:8::9:12 and 12:16::18:24--to frame its future *tetrachords* (sets of four consecutive strings) in ratios of 3:4. These numbers define the only “fixed” tones in tuning theory, and are applicable both to Spiral Fifths and to Just Tuning which furnish the two optional “moveable sounds” within each tetrachord. David organizes all the Levite musicians by lot into 24 groups. And he also provides for 24 “gatekeepers” to be on duty daily in specific locations (I Chronicles 24-26).

Further emphasis on 24 arises in *Revelation* which closes the New Testament with “24 angels with harps” surrounding the celestial throne. Because the octave was conceived as 6 whole tones, divisible into 12 semitones which were further divisible into quartertones, factorial 4=24 held an important place in Bible musicology whatever the authors of *Sepher Yezirah* may have had in mind.

**9. Factorial 5 (5!) “seals” the Mode of the Torah.**

Factorial 5 meaning  $1 \times 2 \times 3 \times 4 \times 5 = 120$  now provides appropriate integers for the normative MODE OF THE TORAH (in which the text was chanted) across a double octave, 30:60::60:120. Fixed sounds established by factorial 4=24 are now multiplied by 5 to provide appropriate integers for the new “moveable sounds.”<sup>32</sup> This tuning merits close attention as the Davidic tuning employed at the dedication of Solomon’s temple (see Figure 8, below). But we must decode the tuning for ourselves from Bible clues.

In preparation for Solomon’s building of the temple,

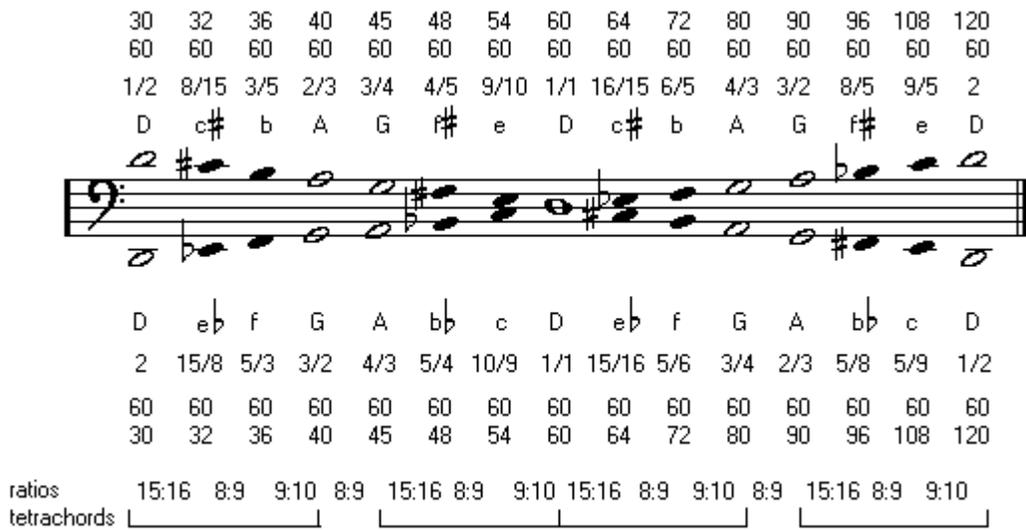
David and the chiefs of the service ... set apart for the service certain of the sons of Asaph, and of Heman, and Jeduthun, who should prophesy with lyres, with harps, and with cymbals ....The number of them along with their brethren, who were trained in singing to the Lord, all who were skillful, was two hundred and eighty eight. And they cast lots for their duties, small and great, teacher and pupil alike (I Chron. 25:1-8).

At the dedication of the temple,

“All of the Levitical singers, Asaph, Heman, and Jeduthun, their sons and kinsmen, arrayed in fine linen, with cymbals, harps, and lyres, stood east of the altar with a hundred and twenty priests who were trumpeters (2 Chron. 5:12).<sup>33</sup>

**Figure 8. Factorial 5 = 120 “seals” the *Mode of the Torah* and/or its reciprocal.**

(Scale numbers are projected against a base-60 reference and modern fractions to clarify pattern.)



Tetrachord frames of 3:4 are filled now by two wholetones of 8:9 and 9:10, preceded in rising order by a semitone of 15:16. This is the normative heptatonic mode of ancient Greece and Israel in the tuning Philo knew as *diatonic*, and the ratios we call *Just*. Reciprocal projections of the ratios show an opposite mode (modern major) falling through the same intervals. When the scales reverse direction, however, their ratios require a new set of defining integers, displayed in Table 2.

6			8	9			12			16	18			24
30	32	36	40	45	48	54	60	64	72	80	90	96	108	120
288	270	240	216	192	180	162	144	135	120	108	96	90	80	72
24			18	16			12			9	8			6

The numbers 288 and 120, by defining the same set of ratios taken in reverse order, confirm the tuning. David’s reorganization was required to provide base-10 substitutes for Mesopotamian base-60 reciprocals. Because all scales freely reverse direction, and these are not self-symmetric, both require both sets of integers. The largest and smallest integers, 288 and 30--joined in reciprocity--sum to 318, the number of “trained men” from his own house with whom Abram rescues nephew Lot from his enemies.<sup>34</sup>

Figure 8 and Table 2 presumably display the tonal resources of singers and trumpeters as they made themselves “heard in unison in praise and thanksgiving to the LORD [YHWH].” With this imaginary performance--for the ancient trumpet was fit only for signaling or for frightening an enemy--a “cloud of glory” then filled the house so that “the priests could not stand to minister ... for the glory of the LORD filled the house of God” (2 Chron. 5:13). Thus in the Bible the power of an imagined Davidic musicology surpasses any priestly ritual. In Philo’s synagogue he experienced the reality.

**10. Factorial 6 (6!) “seals” reciprocal heptatonic modes into a 12-tone system.**

Factorial 6 meaning  $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$  is required to unify the opposite *heptatonic* modes into one self-symmetric 12-tone system.

Figure 9. Factorial 6 = 720 “seals” opposite modes into a linear sequence.



This enlarged *tonal index* is also required for a double octave (180:360::360:720) of the ancient *chromatic genera* which Philo postulates as relevant to Jewish practice. In my notation the rising chromatic tetrachords proceed by two unequal semitones of 15:16 and 24:25 and a minor third of 5:6 (D:eΦ:e:G followed by A:bΦ:b:D).

The integers now available between 360 and 720 also define two alternate pitch classes (shown as C and E, near c and e) which model a related diatonic tetrachord in which the wholetones of 8:9 and 9:10 are reversed (D:C:bΦ:A falling and D:E fΣ:G rising).<sup>35</sup> There are now five tones in Spiral Fifths (C G D A E) to define a nearly universal pentatonic tuning. Developed alone it needs the defining triples 1:3:9:27:81. These are put in scale order by the Chinese rule, “add or subtract one-third”--leading to 81-108-144-96-128 which defines the sequence in self-symmetric scale order, D-E-G-A-C-D rising or D-C-A-G-E-D falling in the numerical sequence 72-81-96-108-128-144.

Heptatonic Spiral Fifths similarly require starting on  $3^6=729$ , and the 12-tone set requires starting on  $3^{11}=177,047$ . Thus the Great Serpent of Spiral Fifths in Figure 4 is

easily tamed in any way one pleases--but never as economically in integers as in Just tuning.

In a tone-circle the radial arms pointing to this pentatonic subset constitute the Egyptian glyph for a star, and the pentagon which frames them seems the likely inspiration for David's pentagonal door to the inner sanctum in the Temple, pictured in Figure 10.<sup>36</sup>

The missing twelfth pitch class now is defined by another integer at  $2^9=8^3=512$  which (under reciprocation) defines either a-flat or g-sharp, approximating the square root of 2, and thus it lies nearly opposite the "seat" of Deity in Figures 5, 6, 7, 10, and 11. Here on the twelfth tone is the only asymmetry in the Just system, and it belongs to the Demiurgic unity, the "1" from which all other numbers arise as multiples, now showing itself as  $2^9=8 \times 8 \times 8=512$ --maturing from Sarah's womb as Isaac, "he laughs." With this Divine Jest we can begin to intuit why Old Testament prophets could say "God is my savior," meaning the living God rather than a future savior, saving the system from within "the midst of his people." In the New

Testament the Savior is *IESOUS*, differentiated from his "Father" (possibly under Greek influence). Early Christian bishops were aware that when read as Greek alphabetical numerals (10.8.200.70.400.200) these six values add up to 888. It would be interesting to know how many Christian authors understood 888 as  $8 \times 8 \times 8=512$ . (The available evidence discourages certainty.)

Philo claims relevance for the ancient *enharmonic genera* in whose tetrachords a ditone of 4:5 is preceded or followed by two quartertones achieved by dividing the 15:16 semitone arithmetically into ratios of 30:31 and 31:32. In smallest integers the ratio sequence is 48:60:62:64 followed by 72:90:93:96 (easily derived from its related chromatic ratios of 48:54:62:64 and 72:81:90:96).<sup>37</sup> To integrate an enharmonic "index" of 96 with the other tunings requires that factorial 6=720 be doubled into 1,440. Jewish folklore credits Adam with the "seeds" of all future generations, and the three Hebrew letters in Adam (אדם) notate 1.4.40. Adam dies at 930, and that just happens to be the smallest integer on which the enharmonic mode can change direction (930:960:992:1240, etc.). Remembering that matrix arithmetic was developed with

Figure 10. Egyptian star glyph

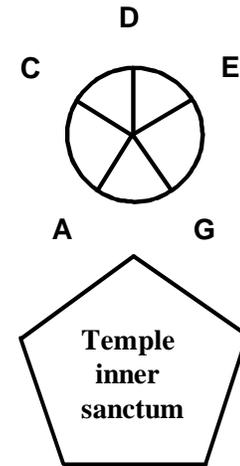
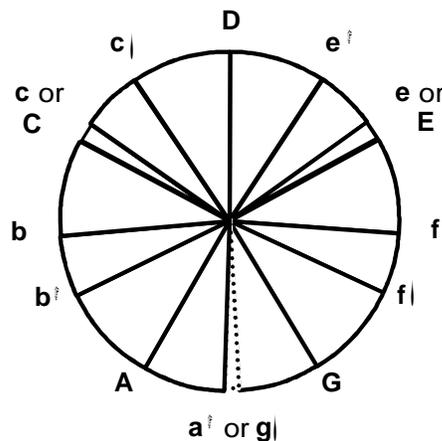


Figure 11. Davidic Just Tuning



“pebble” counters, recall also that the *Gospel of Matthew* declares that “God is able from these stones to raise up children to Abraham”(verse 3:9), an idea repeated at Luke 3:8. During his 40 days of contemplation in the wilderness, Jesus is tempted by the Devil: “If you are the Son of God, command these stones to become loaves of bread” (Matthew 4:3). Thus at factorial  $6=720$  we are approaching the numerical integration of both Testaments.

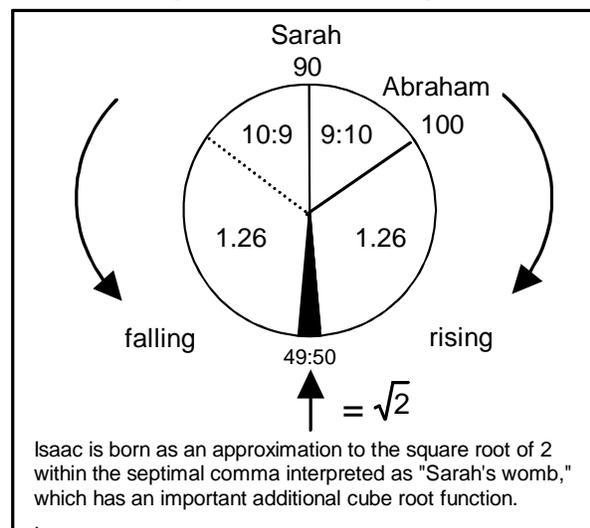
**11. Factorial 7 (7!) provides Divine Oversight.**

Factorial 7 as  $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5,040$  is accessible only to Deity, for “7” is not a “working day.” We noticed earlier how multiplication by 7 integrates the *septimal comma* as the limit of Divine Mercy in the proportion  $35:49::50:70$ , displayed in Figure 5. Its one percent tolerance on each side of the numerically elusive square root of 2 agrees with the “tithe of a tithe” forwarded by the Levites to the High Priest.<sup>38</sup> But another meaning of 7 is also hidden within the construction, and the Bible exposes it by emphasizing the half-sabbatical of  $3 \frac{1}{2}$  years = 42 months = 1,260 days (reducible also to  $7 \times 180$ ) where the cube root of 2 (required for equal temperament) also becomes visible.

In Figure 12 all other detail is suppressed in order to closet Deity (at 7:5) with Abraham and Sarah at  $100:90=10:9$ , treating the *septimal comma* mythically as “Sarah’s womb.” Abraham’s indifference to right and left justifies allowing him an alternate locus. Between Abraham and the *septimal comma* within which Isaac oscillates under reciprocation we discover the ratio  $126:100 = 1.26$ , an excellent approximation to the cube root of 2, correct to the fifth decimal place. It is accessible in integers, however, only under multiplication by 100, and Isaac reaps “hundredfold” whenever he sows. At the index of 5,040 this ratio is present in each semicircle, integrated with Just tuning as defined by factorial  $6 = 720$ . And the half-sabbatical of 1,260 exposes the cube root of 2 to public view.

In ancient Sumerian Uruk, at the dawn of writing, Gilgamesh, literature’s oldest hero, reigns for 126 years. Thus the Jews probably invent little or no arithmetic. I have shown elsewhere how the 1.26 ratio can be developed into a Davidic temperament similar to those of the 16th century AD, a possibility supported by the census of temple workers and by other Bible authors.<sup>39</sup> Here I am interested only in demonstrating the perfect authorial control of the algebra, exercised through the correlation of many tribal tales. A still further subtlety in the arithmetic will be exposed later.

**Figure 12. Divine oversight.**



This preliminary investigation of the Creation week, of Davidic tuning, and of the *Sepher Yezirah*'s factorial analysis is sufficient to indicate that we are on the right general track in studying *the Chosen People* via Philo's musical clues. Authorial control of the numerology never wavers: every numerical ploy leads to further insight. We have gained good reason now to look even more closely at the alternate names of the Deity, where modern Christian scholarship is seriously confused by a profound misunderstanding--attributing different names to different authorial traditions.<sup>40</sup>

**12. How divine names encode Davidic algebra.**

No oriental despot displays more concern with his own name, "My holy name!" than the God of the Jews, and for the very good reason that the Hebrew letters in his various names encode the tonal algebra for his favored musical models. This claim would be preposterous if the Hebrew for God, YHWH, El Shaddai (God Almighty, or the "Mountain" God), and LORD did not possess a powerful unity in the arithmetical evidence reviewed above. Most Semitists have cautiously assumed that the Jews did not possess a numerical notation for the integers until they learned to imitate Greek examples in the last century or so BC. Bible quantification is thus assumed to be restricted to number words rather than number symbols, a practice which protects accuracy in transmission, for some letters are easily confused. However, certain names which appear frequently--those of the Deity and of Abram/Abraham, Sarai/Sarah, Moses, David, Joshua, and perhaps a few others--must have been deemed safe enough to function as alphabetical numerals.<sup>41</sup> The evidence is entirely circumstantial, and is invisible except to readers familiar with ancient harmonical encoding. God's own "great" multiplicative power is exponential, indicated by *dynamis* in Greek. Jewish translators of the Greek *Septuaginta* in the third century BC did not hesitate to use *dynamis* at least 85 times. The following coincidences between names and numbers seem frequent enough to merit attention.

**13. God, *Elohim*.**

The Creator in the first chapter of *Genesis*, translated as God, singular, is *Elohim*,  the Semitic plural for gods, used approximately a thousand times in the Old Testament. Its Hebrew letters can be read (from right to left) as a notation for the numbers (reading left to right) 1.30.5.10.40 all of which play important roles in Bible harmonics. But they also sum to 86, and when it is read as 8<sup>6</sup> meaning 8x8x8x8x8x8 = 262,144, embodying God's "great" exponential power, it "seals" twelve tones from Figure 4 into Spiral Fifths tuning as shown in Table 3, generated via musical fifths of 2:3. (Each multiplication by 3 defines a new pitch class, and so the eleventh new tone is defined by 3<sup>11</sup>=177,147). It seems possible that translators intended *dynamis* to appear 86 times.

In Table 3 the "Holy Land" is aligned according to Abram's perspective, for being indifferent to right and left he belongs in the middle on pitch class "D." In Hebrew Abram is  notating 1.2.200.40 which sum to 3<sup>5</sup>=243. From his perspective, under reciprocation 3<sup>11</sup>=177,147 defines *either* A-flat *or* G-sharp (the 1st or 13th tones), but

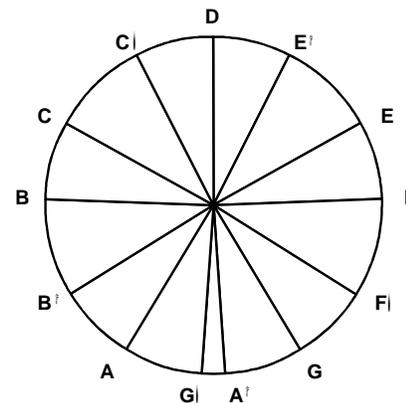
never both together, thus preventing the comma between from confusing each other's tonal function.

**Table 3. When Spiral Fifths tuning is centered on  $3^5=243$ , the twelfth value defines a 13th tone.**

AΦ	EΦ	BΦ	F	C	G	D	A	E	B	FΣ	CΣ	GΣ
$3^{11}$	$3^{10}$	$3^9$	$3^8$	$3^7$	$3^6$	$3^5$	$3^4$	$3^3$	$3^2$	$3^1$	$3^0=1$	
or (under reciprocation)												
	$3^0=1$	$3^1$	$3^2$	$3^3$	$3^4$	$3^5$	$3^6$	$3^7$	$3^6$	$3^9$	$3^{10}$	$3^{11}$

Abram fathers Ishmael, born to Sarai's maid Hagar, and destined to make Abram the grandfather of 12 Arab princes, at the age of 86-- which I read as an allusion to *Elohim's* "God Power" as  $8^6$ . Later, in preparation for his fathering Isaac on Sarah at 100, God adds an h (𐤇) worth 5 units, ⚙️𐤇𐤍𐤏𐤍, and altering Abram to Abraham; 1.2.200.5.40 now sum to 248, a change possibly motivated by the fact that 2, 4, and 8 iterate the reference pitch and thus reinforce the notion of Isaac as only legitimate heir. In Figure 13 the ratio of D:GΣ and D:AΦ is 729/512 in one direction and 1024/729 in the other, a slightly worse approximation to the square root of 2 than Isaac's 720:512 in Figure 9, and still lying within the *septimal comma* of maximum tolerance, and still "saved" by  $8 \times 8 \times 8 = 512$ , present numerically in both Just Tuning and Spiral Fifths.

Figure 13. Elohim's Spiral Fifths "giants"



Spiral Fifths symbolize the "godlike men" created by *Elohim* on the sixth day of the Creation week from the union of the *divine* 3 (Plutarch's metaphor) with *female doubles* ("daughters of men"), met first in  $2 \times 3 = 6$ . Here are the musical *giants* who must be confronted on entering the Holy Land. The cyclic excess in a *perfect fifth* of 2:3 was too slight to enjoy a numerical correction and is handled symbolically by male circumcision, the only point of doctrinal dispute between Jewish and gentile Christians. In Jewish folklore a few heroes are lucky enough to be "born circumcised," but in the Bible even Abraham has to submit, and at 99.

These interpretations are buttressed on the last page of *Revelation* when New Jerusalem descends from clouds as the cube of 12,000, that is, a city of 1,728,000,000,000. The "head" digits of 1,728

**Table 4. The self-symmetric Mode of the Prophets.**

D	C	B	A	G	F	E	D
D	E	F	G	A	B	C	D
432	486	512	576	648	729	768	864
864	972	1024	1152	1296	1458	1536	1728

viewed separately encode a double octave of the self-symmetric heptatonic *Mode of the Prophets* (see Table 4) in its normative diatonic tuning. The “tail” of nine trailing zeros in our Hindu/Arabic numerals proves just sufficient for the remaining six tones to be defined in integers, for the arithmetic is cannily contrived--as the cube of 12,000-- to produce an ecumenical “Heaven” large enough for the virtuous of all peoples.

**14. Yahweh, YHWH, the tribal tetragrammaton.**

*YHWH* appears first in Chapter 2 of *Genesis* in the expression *Elohim YHWH*, “LORD God.” *YHWH*, , usually indicated by LORD (in small capitals) in English translations, notates 10.5.6.5 and occurs some 6000 times in the Old Testament as the special tribal name of the Jewish deity. When its numerals also are read exponentially as  $10^5 \times 6^5 = 60^5 = 777,600,000$  then *YHWH* encodes the same 12 tones in Spiral Fifths as *Elohim*--and in the same way. Its four letters divide 777,600,000 naturally into “heads” and “tails,” for  $6^5 = 7,776$  is Abram’s  $3^5 = 243$  doubled five times, and multiplication by  $10^5$  makes them “leaders” of a hundred thousand. *Elohim* encodes in base-10; *YHWH* encodes in base-60, carrying additional content.<sup>42</sup>

We have already observed that in base 10 no more than six digits are required for all thirteen pitch classes. Thus *YHWH* as  $60^5$  is strictly an artifact of translation from Babylonian base-60 into the preferred Jewish base-10. This *tetragrammaton*, considered too holy to be spoken, can be viewed as a priestly verbal jest aimed at the enemy and mocking his pantheon, for its titular head was the “do-nothing” Anu/An nicknamed “God 1,” written in cuneiform by pressing the stylis more firmly in the clay. His “do nothing” floating place value as some power of 60 is determined entirely by context (meaning by the number of place values required by other numbers associated with him in some particular problem).<sup>43</sup> Judaism requires him to work around the clock.

Base 60 multiplication tables pair all reciprocal products of 2, 3, and 5 as “Platonic twins,” or Isaiah’s Assyrian “horsemen in pairs” whose appearance signals the defeat of the hated Babylonian conqueror. If  $60 = “1”$  is taken as the reference “D” then triples and their reciprocals (treated as Platonic twins) define musical twelfths (octave plus fifth) in both directions, as in Table 5. (Considerable doubling is required to project smaller numbers into scale order. ) Here the first four pairs of numerical values are taken directly from Neugebauer’s representation of the system.<sup>44</sup> The fifth pair, defining C sharp and E flat, requires a five-place sexagesimal expression, thus the reference  $60 = “1”$  must be translated as  $60^5 = 777,600,000$  to avoid fractions in base 10. Only 12 tones in Spiral Fifths are defined within that limit, but the 13th is defined as the reciprocal of the twelfth number (12,9), as it was under *Elohim* as  $8^6$ .

Table 5. Spiral Fifths Twins in Base 60	
using “1” = 60 as reference “D”	
3 and 20	G and A
9 and 6,40	C and E
27 and 2,13,20	F and B
1,21 and 0,44,26,40	B flat and F sharp
4, 3 and 0,14,48,53,20	E flat and C sharp
12, 9	A flat or G sharp

**15. Lord, Adonai.**

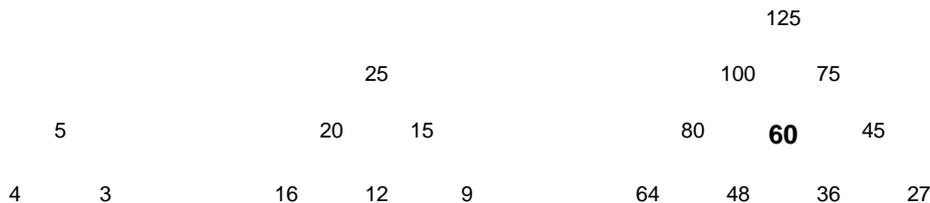
Jews are taught to substitute *Adonai*, the normal Semitic term for Lord, whenever the forbidden *YHWH* appears in the text, and  $\square \blacksquare \Omega \circlearrowright$  notates 1.4.50.10, all of which play major roles in Bible harmonics. But they also sum to 65 which can be read exponentially as  $6^5 = 7,776$  (the “head” digits in *YHWH*), and can be factored as  $2^5 \times 3^5$ . Because  $3^5 = 243$  defines the sixth tone value in Spiral Fifths, under reciprocation 7,776 actually defines all eleven of the symmetries possible in 777,600,000.

**16. God Almighty, El Shaddai.**

*YHWH* insists to Moses that Abraham knew him as *El Shaddai*, “God Almighty” (or the Mountain God), and  $\square \Omega \dagger \bullet \circlearrowright$  notates 1.30 and 300.4.10 which sum to  $31 + 314 = 345$ . This total can be read as  $3 \times 4 \times 5 = 60$ , leading us directly into base 60, or as the Jewish equivalent of Plato’s expression “4:3 mated with the 5,” relevant both to Solomon’s trumpeters and to *YHWH* as  $60^5$ . In Hebrew Moses is  $\approx \dagger \circ$  notating 40.300.5 which also sums to 345, presumably allowing him to talk with God “face to face” or “mouth to mouth.”

Figure 14 presents the triangular matrices associated with expansions of this formula. Here, I suggest, is the “tree of the knowledge of good and evil” in the Garden of Eden. From left to right, ratios are always perfect fourths of 4:3, so that each row constitutes a spiral of fifths. Along the right diagonals  $////$  ratios are 4:5, defining the *ditone* Just major third and functioning as the Babylonian approximation to the cube root of 2. But the progression 60:80:100:125 along the right ascent exposes a shortage of 3 units after 3 successive generations (the double of 64 is 128, not 125), suggesting how “the sins of the fathers are visited on the sons, even to the third and fourth generations,” a decree eventually retracted in *Deuteronomy*.

**Figure 14. El Shaddai, the “Mountain” God**



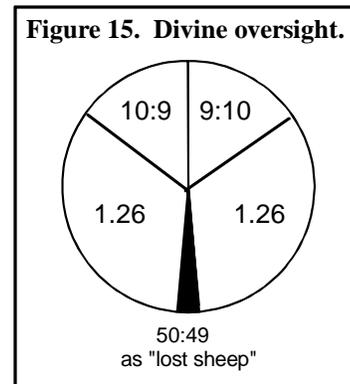
Anyone who studies “God on the Mountain” as  $1=60$  in this matrix might guess that if the 4:5 ratio of 100:125 were corrected to 100:126 (easily done by *adding* one part in a hundred) the cube root of 2 would be better approximated. This is one of the deepest secrets of Jewish harmonics. The solution is equivalent to 1.26 as our equal-tempered major third. But a base of 100 is required to begin with in order to add the correction, and a base of  $100^2 = 10,000$  is required to employ it twice in succession. Here we get a

glimpse into the possible roles of David's fabled "ten thousands," necessary for a *Davidic temperament* which exploits this correction.<sup>45</sup>

By reaching the ratio of 126:100 in this way (by adding 1/100th when also adding 1/4th), multiplication by 7 is avoided in achieving a corrected value for the cube root of 2. Thus *the Chosen People* can be limited to six "working days." But Jewish philosophy, as in the *Sepher Yezirah*, needs *factorial 7* = 7x720=5040 to achieve an overview.

### 17. A Jewish overview.

Figure 15 presents the most significant hidden implications of "7," abstracted from within *factorial 7* = 5,040. The cube root of 2 approximation of 1.26 occurs "naturally" in two symmetric loci (between Abraham and the 7:5 square root approximation), but they are separated at the bottom of the circle by the 49:50 *septimal comma* as a "lost sheep" (with "sins" of one part in a hundred), for it must be joined to the grossly undersized *ditone* of  $(10/9)^2$  at the top if the octave is to be divided into three equal parts.



Proof:  $(10/9)^2 = 100/81$ , and  $(50/49 \times 100/81) = 5,000/3969 = 1.259^+$ , sufficiently accurate to be used with 1.26 in equal temperament. And in this proof, 5000 probably constitutes the hungry assembly fed by the Savior from "seven loaves and a few small fish." (*Matthew* 15:32-38 mentions 4,000 men "besides women and children." *Mark* 6:35-42 mentions 5 loaves and 2 fish feeding 5,000 men, with "twelve baskets full of broken pieces" left over.) This miracle presumably illustrates the Savior's authority to "tread upon serpents and scorpions" (*Luke* 10:19). The numerology of the two *Testaments* is wholly integrated.

But *factorial 7* = 5,040 does not impose any particular musical solution. It presents options. As Figure 15 illustrates, Bible authors and Jewish philosophers alike are content to define problems and tolerances, and in ways which leave alternate procedures open to human choice. Both God and man are conceived as severely disciplined to each other's needs. The effort is to ensure an on-going community.

### 18. Conclusion.

From the evidence presented here it is clear to me that many brilliant literary inventions in the Bible are best understood as tonal-numerical allegory. Authors prove to be perfectly disciplined by the musical and mathematical conventions of the first millennium B.C. before the new Greek sciences altered traditional modes of thinking. The decoding techniques displayed here for the Bible apparently apply also to the numerology of other ancient cultures. Algebraic metaphors, however, even when they are similar, tend to vary from one language to another and force a reader to don a new "thinking cap" when crossing linguistic barriers.

The interpretations offered here have required an unusual degree of interdisciplinary collaboration--made possible by a chain of personal friendships, many with scholars educated abroad and carrying unique perspectives. I am driven by the hope that my own forty years in this "wilderness" of harmonical mythology will help the next generation move ahead with less effort squandered on mistaken dogmas (some of them my own). It is impossible, I believe, for a musician not to be awed both by the technical rigor with which Davidic musicology anticipated the musical values we still live by,<sup>46</sup> and by the Bible's validation of alternate procedures, ensuring human choice. And still more remains to be learned from many sources I could not go into for the sake of maintaining a focussed presentation.

I realize that these musical interpretations of familiar Bible stories may be difficult for some readers to integrate into their own beliefs. All of us suffer from cultural amnesia and always will, for memory cannot retain everything. The historical "re-membering" explored here may be temporarily painful, but I am excited that modern scholarship --in particular regarding ancient Egyptian and Babylonian arithmetic and Assyrian musicology--now brings sufficient insight for us to address anew the content of ancient religious mythology, often passionately damned without the least understanding. And it seems clear that a more "musical" approach to theology should help to relieve some of the confusion of its dogmas, and restore a more equable temper to its discourse.

Good ideas tend to validate themselves eventually by opening further avenues of fruitful discovery. Such adventures are not at an "end" but beckon us onward, encouraging imagination to blossom rather than wither. I realize that I'm not answering old questions about the Bible, but I hope I'm raising some important new ones--with implications for historical and cultural understanding. Jewish law (Torah) is traditionally assumed to have been written by Moses, but if his name (like one of God's) encodes 345 and alludes to a cosmological formula once computed as "4:3 mated with 5" then folk tradition is reassured, Kabbalism earns a certain gratitude for insisting on the importance of symbolic meaning, Bible scholarship can boast of the literal accuracy of its texts, and all of "the people of the Book" should be able to learn to smile on each other.

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## 19. Endnotes.

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<sup>1</sup> Wisdom of Solomon 11:20 (Apocrypha).

<sup>2</sup> My investigation of Bible numerology was inspired by an intense collaboration with Antonio de Nicolas on the tonal numerology of the Rig Veda, India's oldest sacred document. His brilliant seminal study, published as *Meditations Through the Rg Veda: Four-Dimensional Man* (Nicolas Hays, 1976 and 1984), which began as a doctoral thesis (Bangalore: Dharmaram College, 1971), opens all early religious mythology to renewed study of its audial images. Bible arithmetic descends from the related graphic matrices which I develop systematically in *The Myth of Invariance* (Nicolas Hays, 1976 and 1984).

<sup>3</sup> The "Holy Mountain" matrix, now understood as motivating Noah's flood, was the subject of a symposium on "The invention of wisdom" conducted by Harvey Wheeler as editor of the *Journal of Social and Biological Structures*, Volume 5, Number 3, July 1982. His introduction postulates a "mathological musicology in the basic wisdom literatures of several ancient cultures." My essay, "Structure in the ancient wisdom literature: The holy mountain" on pp. 233-248 is followed by several commentaries.

<sup>4</sup> Philo's essays on biblical creationism fill several chapters in his collected works, readily available in *The Works of Philo: New Updated Edition* of the Yonge translation (Hendrickson, 1993), and most of the 12 volumes in the Loeb Classical Library edition. See especially Volume 1 and Supplement I ("Questions and answers on *Genesis*").

<sup>5</sup> Of seminal importance to modern studies is *The Common Background of Greek and Hebrew Civilizations* by Cyrus H. Gordon (Norton, 1965).

<sup>6</sup> Otto Neugebauer, in *The Exact Sciences in Antiquity* (Dover, 1969), offers a simple, clear introduction to Babylonian base-60 calculation, exploiting tables of reciprocals which greatly speeded calculation.

<sup>7</sup> In *Sounds from Silence: Recent Discoveries in Ancient Near Eastern Music* (Berkeley: Bit Enki Publications, 1976, Kilmer, Crocker and Brown document the Greek modes as descending from Assyrian lyre tunings of the second millennium B.C., and tuned (by inference) to the spiral fifths ratios preferred in Greece.

<sup>8</sup> Neil Swerdlow, in *The Babylonian Theory of the Planets* (Princeton, 1998), now emphatically credits the Babylonians for much of the astronomy traditionally credited to the Greeks. His decoding invites serious study.

<sup>9</sup> In his essay "Philo on Music" (*International Journal of Musicology* IX:1, Winter 1991, pp. 124-130), Siegmund Levarie presents the evidence for Philo's empirical knowledge of Alexandrine musical practice.

<sup>10</sup> B.L.van der Waerden, *Science Awakening II: the birth of Astronomy* (Noordhoff, 1974), pp. 64-67.

<sup>11</sup> Harvey Wheeler's unpublished 1969 essay on "The Invention of Political Theory" alerted me to the technical function of musical models, and remains a valuable introduction to this subject.

<sup>12</sup> Rabbi Hertz, whose translation of the Pentateuch is quoted here (from *The Pentateuch and Haftorahs*, Soncino Press, 1973), notes that the author chooses a term whose root emphasizes "blind", non-rational love "for which no reason is to be sought, as it is due solely to the desire of the lover."

<sup>13</sup> Boethius, writing in the 5th century AD, demonstrates his own perfect competence with Greek harmonical models, but encases them in a lamentable pedantry which hides as much as it reveals, useless to Bible scholars who must cope with clever parsimony and a lively humor. *Fundamentals of Music*, transl. by Calin M. Bower (Yale, 1989).

<sup>14</sup> Exodus 18:13-26.

<sup>15</sup> Aristotle's concise characterization of Plato's arithmetic means that--under the principle of *octave equivalence*--integers actually function as *modular residues*. Most Bible examples, and Greek authors other than Plato, need the double octave, 1:2::2:4, and this may justify *Eden* being read in Hebrew as  meaning 70+4+50=124 (from a root meaning delight or pleasure).

<sup>16</sup> A.Z. Idelsohn, in *Jewish Music in Its Historical Development* (Holt, Rinehart and Winston, 1929; Schocken 1975) was able to study the chants of communities still long separated by the Diaspora, looking for clues to ancient custom. "Oriental" affection for variation undoubtedly introduced microintervals, so the Greek forms displayed here should not be considered binding on Jewish practice.

<sup>17</sup> In *The Pythagorean Plato* (Nicolas Hays, 1976 and 1984) I have analyzed Plato's extensive use of these modes.

<sup>18</sup> Isidor Kalisch, *Sepher Yezirah: A book on Creation; or, The Jewish Metaphysics of Remote Antiquity* (New York, 1877; 11th ed. published by Rosae Crucis, 1972), p. 34..

<sup>19</sup> Israel is described as *Jeshurun* at Deut. 32:15 and 33:5 and 26.

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<sup>20</sup> This felicitous description by Robert Brumbaugh in *Plato's Mathematical Imagination* (Chicago University Press, 1954; Kraus Reprint 1968, pp. 221-229), which has not been properly appreciated by classicists, facilitated my further decoding of Plato's musical arithmetic in dialogues other than *Timaeus*.

<sup>21</sup> *Genesis* 41:32.

<sup>22</sup> *Genesis* 4:12-22.

<sup>23</sup> Joseph and Joshua contradict this theory by dying inexplicably at  $10 \times 11 = 110$ . Both men are associated more prominently with other numbers.

<sup>24</sup> Fritz A. Kuttner cites this ancient source in "Prince Chu Tsai Yu's Life and Work," *Ethnomusicology* 19(2):163-204.

<sup>25</sup> It is a very great mystery why this ancient Chinese procedure, attributed to Kuan Tzu c. 700 BC, never found acceptance in the West. It works arithmetically and geometrically, is far simpler than most Western procedures, and applies to Pythagorean matrices.

<sup>26</sup> The last biblical survivor of this race is Og, king of Bashan, whose bedstead measures 4 x 9 cubits (roughly 6 to 13 feet), and his measures take us to the *tetractys* array which sets forth successive musical fifths 2:3. Two in succession require definition by 4:6:9, bounded by the "plane" values of  $2^2$  and  $3^2$ . (Three require 8:12:18:27, etc.)

<sup>27</sup> Louis Ginzberg, *The Legends of the Jews* (Philadelphia, The Jewish Publication Society of America,) 1909-1968. See Index in vol. 7.

<sup>28</sup> No other Greek author confirms Philo's opinion ("On the Creation," par. 96) that the 5:2 ratio "rivals the octave in its consonance." This implies that he is listening to Just musical tenths as consonances, not as arithmetical simplifications.

<sup>29</sup> Neugebauer and Sachs, *Mathematical Cuneiform Texts*, 1945, American Oriental Society, pp. 42-43.

<sup>30</sup> Berossus, the last priest of Marduk, wrote his description of more primitive Babylonian mythology in the third century BC. *Chaldean Account of Genesis*, transl. by George Smith (Minneapolis: Wizards Book Shelf, 1977).

<sup>31</sup> *Exodus* 5:3. (The relevant prime numbers 3 and 5 show up here as Chapter and verse, and  $3 \times 5 = 15$  locates the reference pitch in the middle of the Davidic matrix for the 30:60 octave and its double.)

<sup>32</sup> Greek Dorian in a tuning Ptolemy labels *diatonic syntonon*.

<sup>33</sup> See "Temple Tuning" by McClain with Levarie in *International Journal of Musicology*, Vol. 6., 1997. We have been colleagues since 1954. My Plato book applied his historical and harmonical insight and that of Ernst Levy, now deceased, to Plato's algebraic formulas, and now also to Bible allegories. Progress is a tribute to Levarie's facility in translating fluently from Greek, Latin, Hebrew, Italian, French, and German, to his seminars on musical acoustics where these ideas could be tested, to his friendship with other scholars in philosophy, mathematics, and musicology, and to his continuing interest and editorial assistance. I am drawing here also on our harmonical analysis of the book of *Revelation*, not yet ready for publication. *Tone: A Study in Musical Acoustics* (Kent State University Press, 1968, 1980) co-authored by Levarie and Levy, contains a felicitous introduction to Pythagorean musical thought.

<sup>34</sup> *Genesis* 14:12-16.

<sup>35</sup> Ptolemy credits this alternate diatonic tuning to Didymus, assumed to be a Greek theorist of the 3rd century BC. His name means *twin*. The most accessible English source for Ptolemy's many tunings is Andrew Barker's *Greek Musical Writings II: Harmonic and Acoustic Theory* (Cambridge: Cambridge University Press), 1989.

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<sup>36</sup> I Kings 6:31 in *Revised Standard Version* and *New Revised Standard Version*.

<sup>37</sup> Ptolemy credits these *chromatic* and *enharmonic* tunings also to Didymus. Extant Greek tunings are most accessible now in the translations of Andrew Barker in *Greek Musical Writings II* (Cambridge University Press, 1989).

<sup>38</sup> *Numbers* 18:26-28.

<sup>39</sup> My suggestions for securing a Davidic temperament are presented in "The Star of David as Jewish Harmonical Symbol," *International Journal of Musicology*, Vol. 6, 1997.

<sup>40</sup> The so-called "Documentary hypothesis," argued vehemently for over a century, is summarized in the *Anchor Bible Dictionary* under *Elohism* and *Yahwist*. Jewish scholars protest its sundering of textual unity.

<sup>41</sup> My studies of gematria are indebted to Levarie's collaboration. Initially we were long skeptical of any influence within the Old Testament. The exponential formulas in God names, however, seem convincing to me.

<sup>42</sup> The numerical values in *YHWH* (10.5.6.5) sum to 26, and New Jerusalem's index of 1,728,000,000,000 "seals" a subset of 26 Spiral Fifths (i.e., to  $3^{25}$ ) as a foundation for its matrix. See *The Myth of Invariance*, pp. 113-116.

<sup>43</sup> The numerical nicknames of the Babylonian high gods (60, 50, 40, 30, 20, 15, 10, etc.) have been familiar to Semitists for over a century. They were studied anew by Simo Parpola in "The Assyrian Tree of Life: Tracing the origins of Jewish Monotheism and Greek Philosophy," *Journal of Near Eastern Studies* 52 no. 3, 1993.

<sup>44</sup> Otto Neugebauer, "The Exact Sciences in Antiquity," p. 32. Base 60 reciprocals of successive powers of 3 are successive powers of 20, leading to  $20^3=8,000$  as the volume of the "Holy of Holies" in the Davidic temple. The gematria for *YHWH* merits more detailed attention in another essay, for its matrix conditions tribal history. See Appendix I.

<sup>45</sup> McClain, "The star of David as Jewish harmonical symbol," *International Journal of Musicology* Vol. 6, 1997.

<sup>46</sup> Ernst Levy's *A Theory of Harmony*, published posthumously (SUNY Press, 1985), gives special attention to the natural seventh, temperament, the tonal functions of intervals, etc.